

Combinatorics
Mid-Term Examination
8th March 2013

Instructions: All questions carry equal marks. All sets and collections in the questions are assumed to be finite!

1. Let B_1, \dots, B_n be a collection of subsets of a set S such that union of any l of B_i 's contain at least l elements for any $1 \leq l \leq n$. Prove that we can choose $b_i \in B_i$ such that $b_i \neq b_j$ for all $i \neq j$.
2. Let $m < n$ be positive integers. Show that $m \leq \frac{n}{2}$ is a necessary and sufficient condition for the existence of a Latin square of order n containing a Latin subsquare of order m .
3. Define a Linear Space. Prove that for a linear space with at least two lines, the number of lines is at least as much as the number of points.
4. Define a t -design. Prove that a t -design is also an i -design for all $1 \leq i \leq t$.
5. Define support of a binary codeword. Let C be a perfect binary e -error-correcting code of length n . Let $\mathcal{P} = \{1, 2, \dots, n\}$. Show that \mathcal{P} together with the supports of codewords of weight $2e + 1$ form a $S(e + 1, 2e + 1, n)$ design (with $\lambda = 1$).